Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_3$

$\dot{u}_2 = u_4$

$\dot{u}_3 = f_1\varepsilon^2(1/2\exp(-it)i - 1/2\exp(it)i) + \varepsilon^2(-1/2u_1^3 - 3/10u_3 + 3/10u_4) - 2u_1 + u_2$

$\dot{u}_4 = f_2\varepsilon^2(3/20\exp((-it)/2)u_3 - 3/10\exp((-it)/2)u_4 + 3/20\exp((it)/2)u_3 - 3/10\exp((it)/2)u_4) + u_1 - 2u_2$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 1, i, i\}, \exp(it)\}$

$\vec{e}_2 = \{\{1, 1, -i, -i\}, \exp(-it)\}$

$\vec{z}_1 = \{\{1/4, 1/4, 1/4i, 1/4i\}, \exp(it)\}$

$\vec{z}_2 = \{\{1/4, 1/4, -1/4i, -1/4i\}, \exp(-it)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = f_1\varepsilon^2(3/16\exp(-it)i - 3/16\exp(it)i) + f_2\varepsilon^2(4/55\exp(-it/2)s_2i - 4/25\exp(-3it/2)s_2i - 4/55\exp(it/2)s_1i + 4/25\exp(3it/2)s_1i) + \varepsilon^2(-9/16\exp(-it)s_2^2s_1 + 7/96\exp(-3it)s_2^3 - 9/16\exp(it)s_2s_1^2 + 7/96\exp(3it)s_1^3) + \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon^4)$

$u_2 = f_1\varepsilon^2(-1/16\exp(-it)i + 1/16\exp(it)i) + f_2\varepsilon^2(7/55\exp(-it/2)s_2i + 1/25\exp(-3it/2)s_2i - 7/55\exp(it/2)s_1i - 1/25\exp(3it/2)s_1i) + \varepsilon^2(3/16\exp(-it)s_2^2s_1 - 1/96\exp(-3it)s_2^3 + 3/16\exp(it)s_2s_1^2 - 1/96\exp(3it)s_1^3) + \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon^4)$

$u_3 = f_1\varepsilon^2(1/16\exp(-it) + 1/16\exp(it)) + f_2\varepsilon^2(2/55\exp(-it/2)s_2 - 6/25\exp(-3it/2)s_2 + 2/55\exp(it/2)s_1 - 6/25\exp(3it/2)s_1) + \varepsilon^2(3/16\exp(-it)s_2^2s_1i - 7/32\exp(-3it)s_2^3i - 3/16\exp(it)s_2s_1^2i + 7/32\exp(3it)s_1^3i) - \exp(-it)s_2i + \exp(it)s_1i + O(\varepsilon^4)$

$u_4 = f_1\varepsilon^2(-3/16\exp(-it) - 3/16\exp(it)) + f_2\varepsilon^2(7/110\exp(-it/2)s_2 + 3/50\exp(-3it/2)s_2 + 7/110\exp(it/2)s_1 + 3/50\exp(3it/2)s_1) + \varepsilon^2(-9/16\exp(-it)s_2^2s_1i + 1/32\exp(-3it)s_2^3i + 9/16\exp(it)s_2s_1^2i - 1/32\exp(3it)s_1^3i) - \exp(-it)s_2i + \exp(it)s_1i + O(\varepsilon^4)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = f_1\varepsilon^4(9/64s_2s_1 - 9/128s_1^2 + 3/160i) - 1/8f_1\varepsilon^2 + 93/5500f_2^2\varepsilon^4s_1i + \varepsilon^4(-155/256s_2^2s_1^3i + 9/160s_2s_1^2) + 3/8\varepsilon^2s_2s_1^2i + O(\varepsilon^5)$

$\dot{s}_2 = f_1\varepsilon^4(-9/128s_2^2 + 9/64s_2s_1 - 3/160i) - 1/8f_1\varepsilon^2 - 93/5500f_2^2\varepsilon^4s_2i + \varepsilon^4(155/256s_2^3s_1^2i + 9/160s_2^2s_1) - 3/8\varepsilon^2s_2^2s_1i + O(\varepsilon^5)$