Invariant manifold of your dynamical system

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2:28pm, June 4, 2022

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot u_1 = \varepsilon^2(-a1u_1 + a1u_2) - a2u_4 + u_1 - u_2$$

$$\dot u_2 = \varepsilon^2 a3u_1 - \varepsilon b1u_1u_3 - a4u_4 + \beta^2 u_1 + u_1 - u_2$$

$$\dot u_3 = -\varepsilon^2 a5u_3 + \varepsilon b2u_1u_2$$

$$\dot u_4 = -\varepsilon^2 a7u_4 + \varepsilon a6u_2u_3$$

Invariant subspace basis vectors

$$\vec e_1 = \{\{1, -\beta i + 1, 0, 0\}, \exp(\beta i t)\}$$

$$\vec e_2 = \{\{1, \beta i + 1, 0, 0\}, \exp(-\beta i t)\}$$

$$\vec e_3 = \{\{0, 0, 1, 0\}, \exp(0)\}$$

$$\vec e_4 = \{\{-a2 + a4, -a2\beta^2 - a2 + a4, 0, \beta^2\}, \exp(0)\}$$

$$\vec z_1 = \{\{1/(\beta^2 + 2), (-\beta i + 1)/(\beta^2 + 2), 0, 0\}, \exp(\beta i t)\}$$

$$\vec z_2 = \{\{1/(\beta^2 + 2), (\beta i + 1)/(\beta^2 + 2), 0, 0\}, \exp(-\beta i t)\}$$

$$\vec z_3 = \{\{0, 0, 1, 0\}, \exp(0)\}$$

$\vec{z}_4 = \{\{0, 0, 0, \beta^{-2}\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \exp(-\beta it)s_3s_2\varepsilon(1/2a2a6\beta^5i + 1/2a2a6\beta^4 + 5/2a2a6\beta^3i + 5/2a2a6\beta^2 + 4a2a6\beta i + 2a2a6\beta^{-1}i + 2a2a6\beta^{-2} + 4a2a6 - 1/2a4a6\beta^4 - 5/2a4a6\beta^2 - 2a4a6\beta^{-2} - 4a4a6 + 1/2b1\beta^4 + 1/2b1\beta^3i + 2b1\beta^2 + 2b1\beta i + 2b1\beta^{-1}i + 2b1)/(\beta^6 + 6\beta^4 + 12\beta^2 + 8) + \exp(-\beta it)s_2 + \exp(\beta it)s_3s_1\varepsilon(-1/2a2a6\beta^5i + 1/2a2a6\beta^4 - 5/2a2a6\beta^3i + 5/2a2a6\beta^2 - 4a2a6\beta i - 2a2a6\beta^{-1}i + 2a2a6\beta^{-2} + 4a2a6 - 1/2a4a6\beta^4 - 5/2a4a6\beta^2 - 2a4a6\beta^{-2} - 4a4a6 + 1/2b1\beta^4 - 1/2b1\beta^3i + 2b1\beta^2 - 2b1\beta i - 2b1\beta^{-1}i + 2b1)/(\beta^6 + 6\beta^4 + 12\beta^2 + 8) + \exp(\beta it)s_1 + s_4s_3\varepsilon(a2^2a6\beta^{-2} + a2^2a6 - a2a4a6\beta^{-2} - a2b1\beta^{-2} + a4b1\beta^{-2}) + s_4(-a2 + a4) + O(\varepsilon^2)$

$u_2 = \exp(-\beta it)s_3s_2\varepsilon(-a2a6\beta^{-1}i - 1/2a2a6\beta^{-2} + 1/2a2a6 + 1/2a4a6\beta^{-1}i + 1/2a4a6\beta^{-2} - 1/2b1\beta^{-1}i)/(\beta^2 + 2) + \exp(-\beta it)s_2(\beta i + 1) + \exp(\beta it)s_3s_1\varepsilon(a2a6\beta^{-1}i - 1/2a2a6\beta^{-2} + 1/2a2a6 - 1/2a4a6\beta^{-1}i + 1/2a4a6\beta^{-2} + 1/2b1\beta^{-1}i)/(\beta^2 + 2) + \exp(\beta it)s_1(-\beta i + 1) + s_4s_3\varepsilon(a2a4a6\beta^{-2} + a2a4a6 - a2b1\beta^{-2} - a4^2a6\beta^{-2} + a4b1\beta^{-2}) + s_4(-a2\beta^2 - a2 + a4) + O(\varepsilon^2)$

$u_3 = \exp(-\beta it)s_4s_2\varepsilon(-a2b2\beta i - 2a2b2\beta^{-1}i + a2b2 + 2a4b2\beta^{-1}i - a4b2) + \exp(-2\beta it)s_2^2\varepsilon(1/2b2\beta^{-1}i - 1/2b2) + \exp(\beta it)s_4s_1\varepsilon(a2b2\beta i + 2a2b2\beta^{-1}i + a2b2 - 2a4b2\beta^{-1}i - a4b2) + \exp(2\beta it)s_1^2\varepsilon(-1/2b2\beta^{-1}i - 1/2b2) + s_3 + O(\varepsilon^2)$

$u_4 = \exp(-\beta it)s_3s_2\varepsilon(a6\beta^{-1}i - a6) + \exp(\beta it)s_3s_1\varepsilon(-a6\beta^{-1}i - a6) + s_4\beta^2 + O(\varepsilon^2)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = s_4^2s_1\varepsilon^2(1/2a2^3a6b2\beta^8 - 1/4a2^3a6b2\beta^7i + 4a2^3a6b2\beta^6 - 5/2a2^3a6b2\beta^5i + 13a2^3a6b2\beta^4 - 35/4a2^3a6b2\beta^3i + 43/2a2^3a6b2\beta^2 - 27/2a2^3a6b2\beta i - 9a2^3a6b2\beta^{-1}i + 6a2^3a6b2\beta^{-2} - 2a2^3a6b2\beta^{-3}i +$

$18a2^3a6b2 + 1/4a2^2a4a6b2\beta^7i - 3/4a2^2a4a6b2\beta^6 + 3a2^2a4a6b2\beta^5i - 25/4a2^2a4a6b2\beta^4 + 51/4a2^2a4a6b2\beta^3i - 19a2^2a4a6b2\beta^2 + 49/2a2^2a4a6b2\beta i + 21a2^2a4a6b2\beta^{-1}i - 12a2^2a4a6b2\beta^{-2} + 6a2^2a4a6b2\beta^{-3}i - 25a2^2a4a6b2 - 3/4a2^2b1b2\beta^6 + a2^2b1b2\beta^5i - 19/4a2^2b1b2\beta^4 + 11/2a2^2b1b2\beta^3i - 23/2a2^2b1b2\beta^2 + 10a2^2b1b2\beta i + 6a2^2b1b2\beta^{-1}i - 6a2^2b1b2\beta^{-2} - 13a2^2b1b2 - 1/4a2a4^2a6b2\beta^6 - 1/2a2a4^2a6b2\beta^5i - a2a4^2a6b2\beta^4 - 17/4a2a4^2a6b2\beta^3i + 1/2a2a4^2a6b2\beta^2 - 25/2a2a4^2a6b2\beta i - 15a2a4^2a6b2\beta^{-1}i + 6a2a4^2a6b2\beta^{-2} - 6a2a4^2a6b2\beta^{-3}i + 6a2a4^2a6b2 + 3/4a2a4b1b2\beta^6 - 3/2a2a4b1b2\beta^5i + 6a2a4b1b2\beta^4 - 9a2a4b1b2\beta^3i + 18a2a4b1b2\beta^2 - 18a2a4b1b2\beta i - 12a2a4b1b2\beta^{-1}i + 12a2a4b1b2\beta^{-2} + 24a2a4b1b2 + 1/4a4^3a6b2\beta^4 + 1/4a4^3a6b2\beta^3i + a4^3a6b2\beta^2 + 3/2a4^3a6b2\beta i + 3a4^3a6b2\beta^{-1}i + 2a4^3a6b2\beta^{-3}i + a4^3a6b2 + 1/2a4^2b1b2\beta^5i - 5/4a4^2b1b2\beta^4 + 7/2a4^2b1b2\beta^3i - 13/2a4^2b1b2\beta^2 + 8a4^2b1b2\beta i + 6a4^2b1b2\beta^{-1}i - 6a4^2b1b2\beta^{-2} - 11a4^2b1b2)/(\beta^6 + 6\beta^4 + 12\beta^2 + 8) + s_3^2s_1\varepsilon^2(3/8a2^2a6^2\beta^{-1}i + 3/4a2^2a6^2\beta^{-3}i + 3/8a2^2a6^2\beta^{-5}i - 1/2a2a4a6^2\beta^{-2} - 3/4a2a4a6^2\beta^{-3}i - 1/2a2a4a6^2\beta^{-4} - 3/4a2a4a6^2\beta^{-5}i + 1/2a2a6b1\beta^{-4} - 1/8a4^2a6^2\beta^{-3}i + 1/2a4^2a6^2\beta^{-4} + 3/8a4^2a6^2\beta^{-5}i + 1/4a4a6b1\beta^{-3}i - 1/2a4a6b1\beta^{-4} - 1/8b1^2\beta^{-3}i) + s_3s_1\varepsilon(1/2a2a6\beta^{-2} + 1/2a2a6 + 1/2a4a6\beta^{-1}i - 1/2a4a6\beta^{-2} - 1/2b1\beta^{-1}i) + s_2s_1^2\varepsilon^2(1/4a2a6b2\beta^6 + 5/4a2a6b2\beta^5i + 7/4a2a6b2\beta^4 + 31/4a2a6b2\beta^3i + 11/2a2a6b2\beta^2 + 37/2a2a6b2\beta i + 21a2a6b2\beta^{-1}i + 6a2a6b2\beta^{-2} + 10a2a6b2\beta^{-3}i + 9a2a6b2 + 1/4a4a6b2\beta^5i - 1/2a4a6b2\beta^4 + 1/4a4a6b2\beta^3i - 4a4a6b2\beta^2 - 9/2a4a6b2\beta i - 13a4a6b2\beta^{-1}i - 8a4a6b2\beta^{-2} - 10a4a6b2\beta^{-3}i - 10a4a6b2 + 1/4b1b2\beta^5i - 3/4b1b2\beta^4 + 5/2b1b2\beta^3i - 5/2b1b2\beta^2 + 7b1b2\beta i + 6b1b2\beta^{-1}i + 2b1b2\beta^{-2} - b1b2)/(\beta^6 + 6\beta^4 + 12\beta^2 + 8) + s_1\varepsilon^2(-1/2a1\beta i - 1/2a1 + 1/2a3\beta^{-1}i) + O(\varepsilon^3)$

$\dot{s}_2 = s_4^2s_2\varepsilon^2(1/2a2^3a6b2\beta^8 + 1/4a2^3a6b2\beta^7i + 4a2^3a6b2\beta^6 + 5/2a2^3a6b2\beta^5i + 13a2^3a6b2\beta^4 + 35/4a2^3a6b2\beta^3i + 43/2a2^3a6b2\beta^2 + 27/2a2^3a6b2\beta i + 9a2^3a6b2\beta^{-1}i + 6a2^3a6b2\beta^{-2} + 2a2^3a6b2\beta^{-3}i + 18a2^3a6b2 - 1/4a2^2a4a6b2\beta^7i - 3/4a2^2a4a6b2\beta^6 - 3a2^2a4a6b2\beta^5i - 25/4a2^2a4a6b2\beta^4 - 51/4a2^2a4a6b2\beta^3i - 19a2^2a4a6b2\beta^2 - 49/2a2^2a4a6b2\beta i - 21a2^2a4a6b2\beta^{-1}i - 12a2^2a4a6b2\beta^{-2} - 6a2^2a4a6b2\beta^{-3}i - 25a2^2a4a6b2 - 3/4a2^2b1b2\beta^6 - a2^2b1b2\beta^5i - 19/4a2^2b1b2\beta^4 - 11/2a2^2b1b2\beta^3i - 23/2a2^2b1b2\beta^2 - 10a2^2b1b2\beta i - 6a2^2b1b2\beta^{-1}i - 6a2^2b1b2\beta^{-2} - 13a2^2b1b2 - 1/4a2a4^2a6b2\beta^6 + 1/2a2a4^2a6b2\beta^5i - a2a4^2a6b2\beta^4 + 17/4a2a4^2a6b2\beta^3i + 1/2a2a4^2a6b2\beta^2 + 25/2a2a4^2a6b2\beta i + 15a2a4^2a6b2\beta^{-1}i + 6a2a4^2a6b2\beta^{-2} + 6a2a4^2a6b2\beta^{-3}i + 6a2a4^2a6b2 + 3/4a2a4b1b2\beta^6 + 3/2a2a4b1b2\beta^5i + 6a2a4b1b2\beta^4 + 9a2a4b1b2\beta^3i + 18a2a4b1b2\beta^2 + 18a2a4b1b2\beta i + 12a2a4b1b2\beta^{-1}i +$

$12a2a4b1b2\beta^{-2} + 24a2a4b1b2 + 1/4a4^3a6b2\beta^4 - 1/4a4^3a6b2\beta^3i + a4^3a6b2\beta^2 - 3/2a4^3a6b2\beta i - 3a4^3a6b2\beta^{-1}i - 2a4^3a6b2\beta^{-3}i + a4^3a6b2 - 1/2a4^2b1b2\beta^5i - 5/4a4^2b1b2\beta^4 - 7/2a4^2b1b2\beta^3i - 13/2a4^2b1b2\beta^2 - 8a4^2b1b2\beta i - 6a4^2b1b2\beta^{-1}i - 6a4^2b1b2\beta^{-2} - 11a4^2b1b2)/(\beta^6 + 6\beta^4 + 12\beta^2 + 8) + s_3^2s_2\varepsilon^2(-3/8a2^2a6^2\beta^{-1}i - 3/4a2^2a6^2\beta^{-3}i - 3/8a2^2a6^2\beta^{-5}i - 1/2a2a4a6^2\beta^{-2} + 3/4a2a4a6^2\beta^{-3}i - 1/2a2a4a6^2\beta^{-4} + 3/4a2a4a6^2\beta^{-5}i + 1/2a2a6b1\beta^{-4} + 1/8a4^2a6^2\beta^{-3}i + 1/2a4^2a6^2\beta^{-4} - 3/8a4^2a6^2\beta^{-5}i - 1/4a4a6b1\beta^{-3}i - 1/2a4a6b1\beta^{-4} + 1/8b1^2\beta^{-3}i) + s_3s_2\varepsilon(1/2a2a6\beta^{-2} + 1/2a2a6 - 1/2a4a6\beta^{-1}i - 1/2a4a6\beta^{-2} + 1/2b1\beta^{-1}i) + s_2^2s_1\varepsilon^2(1/4a2a6b2\beta^6 - 5/4a2a6b2\beta^5i + 7/4a2a6b2\beta^4 - 31/4a2a6b2\beta^3i + 11/2a2a6b2\beta^2 - 37/2a2a6b2\beta i - 21a2a6b2\beta^{-1}i + 6a2a6b2\beta^{-2} - 10a2a6b2\beta^{-3}i + 9a2a6b2 - 1/4a4a6b2\beta^5i - 1/2a4a6b2\beta^4 - 1/4a4a6b2\beta^3i - 4a4a6b2\beta^2 + 9/2a4a6b2\beta i + 13a4a6b2\beta^{-1}i - 8a4a6b2\beta^{-2} + 10a4a6b2\beta^{-3}i - 10a4a6b2 - 1/4b1b2\beta^5i - 3/4b1b2\beta^4 - 5/2b1b2\beta^3i - 5/2b1b2\beta^2 - 7b1b2\beta i - 6b1b2\beta^{-1}i + 2b1b2\beta^{-2} - b1b2)/(\beta^6 + 6\beta^4 + 12\beta^2 + 8) + s_2\varepsilon^2(1/2a1\beta i - 1/2a1 - 1/2a3\beta^{-1}i) + O(\varepsilon^3)$

$\dot{s}_3 = s_4^2s_3\varepsilon^2(-a2^3a6b2\beta^2 - a2^3a6b2\beta^{-2} - 2a2^3a6b2 + a2^2a4a6b2\beta^{-2} + a2^2a4a6b2 + 2a2^2b1b2\beta^{-2} + a2^2b1b2 + a2a4^2a6b2\beta^{-2} + a2a4^2a6b2 - 4a2a4b1b2\beta^{-2} - a2a4b1b2 - a4^3a6b2\beta^{-2} + 2a4^2b1b2\beta^{-2}) + s_4^2\varepsilon(a2^2b2\beta^2 + a2^2b2 - a2a4b2\beta^2 - 2a2a4b2 + a4^2b2) + s_3s_2s_1\varepsilon^2(a2a6b2\beta^2 + 3a2a6b2 - a4a6b2 + 2b1b2)/(\beta^2 + 2) - s_3\varepsilon^2a5 + 2s_2s_1\varepsilon b2 + O(\varepsilon^3)$

$\dot{s}_4 = s_4s_3^2\varepsilon^2(a2a4a6^2\beta^{-2} + a2a4a6^2\beta^{-4} - a2a6b1\beta^{-4} - a4^2a6^2\beta^{-4} + a4a6b1\beta^{-4}) + s_4s_3\varepsilon(-a2a6\beta^{-2} - a2a6 + a4a6\beta^{-2}) + s_4s_2s_1\varepsilon^2(-2a2a6b2\beta^{-2} - 2a2a6b2 + 2a4a6b2\beta^{-2}) - s_4\varepsilon^2a7 + O(\varepsilon^3)$