Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

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The specified dynamical system

$\dot u_1 = -\varepsilon u_1^2 - u_1 + u_2$

$\dot u_2 = \varepsilon u_2^2 + u_1 - u_2$

Invariant subspace basis vectors

$\vec e_1 = \{\{1, 1\}, \exp(0)\}$

$\vec z_1 = \{\{1/2, 1/2\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = 3/8\varepsilon^3 s_1^4 - 1/2\varepsilon s_1^2 + O(\varepsilon^4) + s_1$

$u_2 = -3/8\varepsilon^3 s_1^4 + 1/2\varepsilon s_1^2 + O(\varepsilon^4) + s_1$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = -3/4\varepsilon^4 s_1^5 + \varepsilon^2 s_1^3 + O(\varepsilon^5)$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$z_{11} = 3/2\varepsilon^4 s_1^4 + 3/4\varepsilon^3 s_1^3 - 1/2\varepsilon^2 s_1^2 - 1/2\varepsilon s_1 + O(\varepsilon^5) + 1/2$

$z_{12} = 3/2\varepsilon^4 s_1^4 - 3/4\varepsilon^3 s_1^3 - 1/2\varepsilon^2 s_1^2 + 1/2\varepsilon s_1 + O(\varepsilon^5) + 1/2$